Trigonometrische Formeln

1.) $$\sin^2 p + \cos^2 p = 1; \quad \text{tang}\, p \cdot \text{ctg}\, p = 1; \quad \sec p \cdot \cos p = 1$$ (T-1)

2.) $$\text{cosec}\, p \cdot \sin p = 1, \quad \sec^2 p - \text{tg}^2 p = 1; \quad \text{cosec}^2 p - \text{cotg}^2 p = 1$$ (T-2)

3.) $$\sin \text{vers}\, p + \cos p = 1; \quad \cos \text{vers}\, p + \sin p = 1$$ (T-3)

4.) $$\sin p = \sqrt{1-\cos^2 p} = \frac{\text{tg}\, p}{\sqrt{1+\text{tg}^2 p}} = \frac{1}{\sqrt{1+\text{ctg}^2 p}} =$$ (T-4)

$$\frac{\sqrt{\sec^2 p - 1}}{\sec p} = \frac{1}{\text{cosec}\, p} = \cos p\, \text{tg}\, p$$ -

5.) $$\cos p = \sqrt{1-\sin^2 p} = \frac{1}{\sqrt{1+\text{tg}^2 p}} = \frac{\text{ctg}\, p}{\sqrt{1+\text{ctg}^2 p}} =$$ (T-5)

$$\frac{1}{\sec p} = \frac{\sqrt{\text{cosec}^2 p - 1}}{\text{cosec}\, p} = \sin p \cdot \text{ctg}\, p.$$

6.) $$\text{tg}\, p = \frac{\sin p}{\sqrt{1-\sin^2 p}} = \frac{\sqrt{1-\cos^2 p}}{\cos p} = \frac{1}{\text{cotg}\, p} =$$ (T-6)

$$\sqrt{\sec^2 p - 1} = \frac{1}{\sqrt{\text{cosec}^2 - 1}}.$$

7.) $$\text{cotg}\, p = \frac{\sqrt{1-\sin^2 p}}{\sin p} = \frac{\cos p}{\sqrt{1-\cos^2 p}} = \frac{1}{\text{tang}\, p} =$$ (T-7)

$$\frac{1}{\sqrt{\sec^2 p - 1}} = \sqrt{\text{cosec}^2 p - 1}.$$

8.) $$\sec p = \frac{1}{\sqrt{1+\sin^2 p}} = \frac{1}{\cos p} = \sqrt{1+\text{tg}^2 p} =$$ (T-8)

$$\frac{\sqrt{1+\text{ctg}^2 p}}{\text{ctg}\, p} = \frac{\text{cosec}\, p}{\sqrt{\text{cosec}^2 p - 1}} = \frac{\text{tg}\, p}{\sin p}.$$

9.) $$\text{cosec}\, p = \frac{1}{\sin p} = \frac{1}{\sqrt{1-\cos^2 p}} = \frac{\sqrt{1+\text{tg}^2 p}}{\text{tg}\, p} =$$ (T-9)

$$\sqrt{1+\operatorname{ctg}^2 p} = \frac{\sec p}{\sqrt{\sec^2 p - 1}} = \frac{\operatorname{ctg} p}{\cos p}.$$

10.) $$\sin\operatorname{vers} p = 1 - \cos p = 2\sin^2 \frac{p}{2}$$ (T-10)

11.) $$\cos\operatorname{vers} p = 1 - \sin p = 2\sin^2(45° - \frac{p}{2})$$ (T-11)

12.) $$\sin\frac{p}{2} = \sqrt{\frac{1}{2} - \frac{1}{2}\cos p}\,; \quad \cos\frac{1}{2}p = \sqrt{\frac{1}{2} + \frac{1}{2}\cos p}$$ (T-12)

13.) $$\operatorname{tg}\frac{1}{2}p = \frac{\sin p}{1+\cos p}; \quad \operatorname{cotg}\frac{1}{2}p = \frac{\sin p}{1-\cos p}.$$ (T-13)

14.) $$\sin p + \cos p = \sqrt{1+\sin 2p} = \cos(45 - p)\cdot\sqrt{2}$$ (T-14)

15.) $$\cos p - \sin p = \sqrt{1-\sin 2p} = \sin(45° - p)\cdot\sqrt{2}$$ (T-15)

16.) $$\operatorname{tg} p + \operatorname{ctg} p = 2\operatorname{cosec} 2p; \quad \operatorname{ctg} p - \operatorname{tg} p = 2\operatorname{ctg} 2p$$ (T-16)

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Trigonometrische Formeln

17.) $$1+\sin p = 2\sin^2(45° + \frac{1}{2}p) \quad 1-\sin p = 2\sin^2(45° - \frac{1}{2}p)$$ (T-17)

18.) $$\frac{1+\sin p}{1-\sin p} = \operatorname{tg}^2(45° + \frac{1}{2}p) \quad \frac{1+\sin p}{\cos p} = \operatorname{tg}^2(45° + \frac{1}{2}p)$$ (T-18)

19.) $$\frac{1+\operatorname{tg} p}{1-\operatorname{tg} p} = \operatorname{tg}(45° + p) \quad \frac{1-\operatorname{tg} p}{1+\operatorname{tg} p} = \operatorname{tg}(45° - p)$$ (T-19)

20.) $$\sin(30° + p) = \cos p - \sin(30° - p)$$ (T-20)

21.) $$\cos(30° + p) = \cos(30° - p) - \sin p$$ (T-21)

22) $$\sin(60° - p) = \sin(60° + p) - \sin p$$ (T-22)

23) $$\cos(60° - p) = \cos p - \cos(60° + p)$$ (T-23)

24.) $$2\sin^2 p = 1 - \cos 2p$$ (T-24)

$$4\sin^3 p = 3\sin p - \sin 3p$$

$$8\sin^4 p = \cos 4p - 4\cos 2p + 3$$

$$16\sin^5 p = \sin 5p - 5\sin 3p + 10\sin p$$

25) $$2\cos^2 p = 1 + \cos 2p$$ (T-25)
$$4\cos^3 p = \cos 3p + 3\cos p$$
$$8\cos^4 p = \cos 4p + 4\cos 2p + 3$$
$$16\cos^5 p = \cos 5p - 5\cos 3p + 10\cos p$$

26.) $$\sin 2p = 2\sin p \cos p$$ (T-26)
$$\sin 3p = 3\sin p \cdot \cos^2 p - \sin^3 p$$
$$\sin 4p = 4\sin p \cdot \cos p(\cos^2 p - \sin^2 p)$$
$$\sin n\, p = \sin(n-1)\, p \cdot \cos p + \cos(n-1)\, p \cdot \sin p$$

27.) $$\cos 2p = \cos^2 p - \sin^2 p = 2\cos^2 p - 1 = 1 - 2\sin^2 p$$ (T-27)
$$\cos 3p = \cos^3 p - 3\sin^2 p \cdot \cos p$$
$$\cos 4p = \cos^4 p - 6\sin^2 p[\cdot]\cos^2 p + \sin^4 p$$
$$\cos n\, p = 2\cos(n-1)p \cdot \cos p - \cos(n-2)\, p$$

B.

28.) $$\sin(p+q) = \sin p \cos q + \cos p \sin q$$ (T-28)

29.) $$\sin(p-q) = \sin p \cos q - \cos p \cdot \sin p$$ (T-29)

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Trigonometrische Formeln

30.) $$\cos(p+q) = \cos p \cdot \cos q - \sin p[\cdot]\sin q$$ (T-30)

31.) $$\cos(p-q) = \cos p \cdot \cos q + \sin p[\cdot]\sin q$$ (T-31)

32.) $$\operatorname{tg}(p+q) = \frac{\operatorname{tg} p + \operatorname{tg} q}{1 - \operatorname{tg} p \cdot \operatorname{tg} q} = \frac{\operatorname{ctg} p + \operatorname{ctg} q}{\operatorname{ctg} p \cdot \operatorname{ctg} q - 1}$$ (T-32)

33.) $$\operatorname{tg}(p-q) = \frac{\operatorname{tg} p - \operatorname{tg} q}{1 + \operatorname{tg} p \cdot \operatorname{tg} q} = \frac{\operatorname{ctg} q - \operatorname{ctg} p}{\operatorname{ctg} p[\cdot]\operatorname{ctg} q + 1}$$ (T-33)

34.) $$\operatorname{ctg}(p+q) = \frac{\operatorname{ctg} p \cdot \operatorname{ctg} q - 1}{\operatorname{ctg} p + \operatorname{ctg} q} = \frac{1 - \operatorname{tg} p[\cdot]\operatorname{tg} q}{\operatorname{tg} p + \operatorname{tg} q}$$ (T-34)

35.) $$\operatorname{ctg}(p-q) = \frac{\operatorname{ctg} p[\cdot]\operatorname{ctg} q + 1}{\operatorname{ctg} q - \operatorname{ctg} p} = \frac{1 + \operatorname{tg} p \cdot \operatorname{tg} q}{\operatorname{tg} p - \operatorname{tg} q}$$ (T-35)

36.) $$\sin p\cdot\sin q=\frac{1}{2}\cos(p-q)-\frac{1}{2}\cos(p+q)$$ (T-36)

37.) $$\cos p\cdot\cos q=\frac{1}{2}\cos(p-q)+\frac{1}{2}\cos(p+q)$$ (T-37)

38.) $$\sin p\cdot\cos q=\frac{1}{2}\sin(p+q)+\frac{1}{2}\sin(p-q)$$ (T-38)

39.) $$\cos p\cdot\sin q=\frac{1}{2}\sin(p+q)-\frac{1}{2}\sin(p-q)$$ (T-39)

40.) $$\sin p+\sin q=2\sin\frac{1}{2}(p+q)\cdot\cos\frac{1}{2}(p-q)$$ (T-40)

41.) $$\sin p-\sin q=2\cos\frac{1}{2}(p+q)\cdot\sin\frac{1}{2}(p-q)$$ (T-41)

42.) $$\cos p+\cos q=2\cos\frac{1}{2}(p+q)[\cdot]\cos\frac{1}{2}(p-q)$$ (T-42)

43.) $$\cos q-\cos p=2\sin\frac{1}{2}(p+q)\cdot\sin\frac{1}{2}(p-q)$$ (T-43)

44.) $$\operatorname{tg} p+\operatorname{tg} q=\frac{\sin(p+q)}{\cos p\cdot\cos q};\qquad \operatorname{tg} p-\operatorname{tg} q=\frac{\sin(p-q)}{\cos p\cdot\cos q}$$ (T-44)

45.) $$\cot \mathrm{p}+\cot \mathrm{q}=\frac{\sin(\mathrm{p}+\mathrm{q})}{\sin \mathrm{p}\cdot\sin \mathrm{q}};\qquad \cot \mathrm{p}-\cot \mathrm{q}=\frac{\sin(\mathrm{q}-\mathrm{p})}{\sin \mathrm{p}\cdot\sin \mathrm{q}}$$ (T-45)

46.) $$\cot \mathrm{p}+\operatorname{tg} \mathrm{q}=\frac{\cos(\mathrm{p}-\mathrm{q})}{\sin \mathrm{p}[\cdot]\cos \mathrm{q}};\qquad \cot \mathrm{p}-\operatorname{tg} \mathrm{q}=\frac{\cos(\mathrm{p}+\mathrm{q})}{\sin \mathrm{p}\cdot\cos \mathrm{q}}$$ (T-46)

47.) $$\frac{\sin p+\sin q}{\sin p-\sin q}=\frac{\operatorname{tg}\frac{1}{2}(p+q)}{\operatorname{tg}\frac{1}{2}(p-q)}=\operatorname{tg}\frac{1}{2}(p+q)[\cdot]\cot\frac{1}{2}(\mathrm{p}-\mathrm{q})$$ (T-47)

48.) $$\frac{\cos p+\cos q}{\cos p-\cos q}=\frac{\cot\frac{1}{2}(\mathrm{p}+\mathrm{q})}{\operatorname{tg}\frac{1}{2}(q-p)}=\cot\frac{1}{2}(\mathrm{p}+\mathrm{q})[\cdot]\cot\frac{1}{2}(\mathrm{q}-\mathrm{p})$$ (T-48)

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Trigonometrische Formeln

49) $$\frac{\sin p+\sin q}{\cos p+\cos q}=\frac{\cos p-\cos q}{\sin q-\sin p}=\operatorname{tg}\frac{1}{2}(p+q)$$ (T-49)

50) $$\frac{\sin p+\sin q}{\cos p-\cos q}=\frac{\cos q+\cos p}{\sin q-\sin p}=\cot\frac{1}{2}(\mathrm{q}-\mathrm{p})$$ (T-50)

51) $$\frac{\operatorname{tg} p+\operatorname{tg} q}{\operatorname{tg} p-\operatorname{tg} q}=\frac{\cot \mathrm{p}+\cot \mathrm{q}}{\cot \mathrm{q}-\cot \mathrm{p}}=\frac{\sin(p+q)}{\sin(p-q)}$$ (T-51)

52) $$\frac{\operatorname{tg} p+\operatorname{tg} q}{\cot \mathrm{p}+\cot \mathrm{q}}=\frac{\operatorname{tg} p-\operatorname{tg} q}{\cot \mathrm{q}-\cot \mathrm{p}}=\operatorname{tg} p\cdot\operatorname{tg} q$$ (T-52)

53) $$\frac{\operatorname{tg} p+\cot \mathrm{q}}{\cot p+\operatorname{tg} q}=\frac{\cot \mathrm{q}-\operatorname{tg} \mathrm{p}}{\cot \mathrm{p}-\operatorname{tg} \mathrm{q}}=\operatorname{tg} p\cdot\cot \mathrm{q}$$ (T-53)

54) $$\frac{\operatorname{tg} p+\operatorname{tg} q}{\cot \mathrm{p}-\operatorname{tg} \mathrm{q}}=\operatorname{tg} p\cdot\operatorname{tg}(p+q)$$ (T-54)

55) $$\frac{\cot \mathrm{p}+\cot \mathrm{q}}{\cot \mathrm{p}-\operatorname{tg} \mathrm{q}}=\cot \mathrm{q}\cdot\operatorname{tg}(\mathrm{p}+\mathrm{q})$$ (T-55)

56) $$\frac{\operatorname{tg} p-\operatorname{tg} q}{\cot \mathrm{p}+\operatorname{tg} \mathrm{q}}=\operatorname{tg} p\cdot\operatorname{tg}(p-q)$$ (T-56)

57) $$\frac{\cot \mathrm{q}-\cot \mathrm{p}}{\operatorname{tg} q+\cot p}=\cot \mathrm{q}\cdot\operatorname{tg}(\mathrm{p}-\mathrm{q})$$ (T-57)

58) $$1+\operatorname{tg} p\cdot\operatorname{tg} q=\frac{\cos(p-q)}{\cos p\cdot\cos q};\quad 1-\operatorname{tg} p\cdot\operatorname{tg} q=\frac{\cos(p+q)}{\cos p\cdot\cos q}$$ (T-58)

59) $$\sin(p+q)\cdot\sin(p-q)=\sin^2 p-\sin^2 q=\frac{1}{2}\cos 2q-\frac{1}{2}cos 2p$$ (T-59)

60) $$\cos(p+q)[\cdot]\cos(p-q)=\cos^2 p-\sin^2 q=\frac{1}{2}\cos 2q-\frac{1}{2}\sin 2p$$ (T-60)

61) $$\sin(p+q)[\cdot]\cos(p-q)=\frac{1}{2}\sin 2p+\frac{1}{2}\sin 2q$$ (T-61)

62) $$\sin(p-q)[\cdot]\cos(p+q)=\frac{1}{2}\sin 2p-\frac{1}{2}\sin 2q$$ (T-62)

$$\frac{\cos mu}{\cos^m u}=(m)_0-(m)_2\operatorname{tang}^2 u+(m)_4\operatorname{tg}^4 u-(m)_6\operatorname{tg}^6 u+\cdots$$ (T-63

$$\frac{\sin mu}{\sin^m u}=(m)_1\operatorname{tg} u-(m)_3\operatorname{tg}^3 u+(m)_5\operatorname{tg}^5 u-\cdots$$ (T-64)

Kettenbrüche

$$\frac{a}{b} = q + \cfrac{1}{q_1 + \cfrac{1}{q_2 + \ddots}} = K(q, q_1 \ldots q_n) \qquad \text{(K-1)}$$

$$\frac{b}{a} = K_1 = \frac{1}{K} = K_1(0, q, q_1 \ldots q_n) \qquad \text{(K-2)}$$

1.) Jeder gem. Bruch ist in einen endlichen Kett.Br. entwickelbar

2.) Jeder K.B. mit endl. Zahl von Theilnehmern ist in einen gewöhnlichen Bruch verwandelbar
K_0, K_1, K_2 ... Näherungsbrüche bis zur 0^{ten}, 1^{ten}, 2^{ten} ... Stelle

3.) $K_s = \dfrac{q_s \cdot Z_{s-1} + Z_{s-2}}{q_s N_{s-1} + N_{s-2}} = \dfrac{Z_s}{N_s}$

$Z_s = q_s\, Z_{s-1} + Z_{s-2}$; $\quad N_s = q_s\, N_{s-1} + N_{s-2}$ (K-3)

4.) $N_s \cdot Z_{s-1} - Z_s \left[\cdot\right] N_{s-1} = (-1)^s$ (K-4)

5.) $K_{s+1} - K_s = \dfrac{(-1)^s}{N_s \cdot N_{s+1}}$; $\quad \left|K_{s+1} - K_s\right| = \dfrac{1}{N_s \cdot N_{s+1}}$ (K-5)

6.) $K_0 < K_1 \quad, K_2 < K_3 \quad, K_4 < K_5 \ldots K_s < K_{s+1}$ s gerade (K-6)

7.) $K_1 > K_2, K_3 > K_4 \ldots K_s > K_{s+1} \cdots s$ ungerade (K-7)

8.) $K_0 < K_2 < K_4 < K_6 \cdots$ (K-8)
$K_1 > K_3 > K_5 > K_7 > \cdots$

od. $K_0 < K_2 < K_1$
$K_2 < K_3 < K_4$
$K_4 < K_5 < K_3$

} Jeder Näherungsbr. liegt zwischen seinen <u>vorausgehenden</u> Näherungsbrüchen

Mathematische Skizze

Fehler $| K_s - K | = f < \dfrac{1}{N_s^2}$ (K-9)

Grenzwerte

Summe: $\lim(\Phi_n \pm \Psi_n) = \Phi \pm \Psi = \lim \Phi_n \pm \lim \Psi_n$ (G-1)

Product: $\lim\left(\Phi_n \cdot \Psi_n\right) = \Phi \cdot \Psi = \lim \Phi_n \cdot \lim \Psi_n$ (G-2)

Quotient: $\lim\left(\frac{\Phi_n}{\Psi_n}\right) = \frac{\Phi}{\Psi} = \frac{\lim \Phi_n}{\lim \Psi_n}$ (G-3)

$\lim c \cdot \Phi_n = c \cdot \lim \Phi_n$ (G-4)

$\lim \log_b \Phi_n = \log_b \lim \Phi_n$ (G-5)

$\lim \Phi_n^{\Psi_n} = \lim \Phi_n^{\lim \Psi_n}$ (G-6)

Wenn: $\lim \frac{\Phi_n}{\lim \Psi_n} = a$ So: $\lim \frac{\Psi_n}{\Phi_n} = \frac{1}{a}$ (G-7)

Allgemein:

$y = f(\varphi(x)) \quad | \quad y = f(z), z = \varphi(x)$

$\lim\limits_{x=x_1} \varphi(x) = \varphi(x_1) = z_1$ wenn $\varphi(x)$ an Stelle $x = x_1$ stetig (G-8)

$\lim\limits_{z=z_1} f(z) = f(z_1)$ wenn $f\left(z\right)$ an der Stelle $z = z_1$ stetig

$= f\left(\lim\limits_{x=x_1} \varphi\left(x\right)\right)$ (G-9)

$\lim\limits_{x=x_1} f\left(\varphi\left(x\right)\right) = f\left(\lim\limits_{x=x_1} \varphi\left(x\right)\right)$ der lim. einer Function = Function des lim (G-10)

Bestimmung

$$\lim_{x=\pm\infty}\left(1+\frac{1}{x}\right)^x = e \quad \text{(G-11)}$$

$$\lim_{x=\pm\infty}\left(1+\frac{a}{x}\right)^x = e^a \qquad \lim_{x\to 0}\frac{x}{\ln\left(1+x\right)} = 1 \quad \text{(G-12)}$$

$$\lim_{x=0}\left(1+ax\right)^{1/x}=e^a \qquad \lim_{x=0}\frac{b^x-1}{x}=\ln b \qquad \text{(G-13)}$$

$$\lim_{x=0}\frac{\log_b\left(1+ax\right)}{x}=\frac{a}{\ln b} \qquad \lim_{x=0}\frac{e^x-1}{x}=1 \qquad \text{(G-13)}$$

$$\lim_{x=0}\frac{\log_b\left(1+x\right)}{x}=\frac{1}{\ln b} \qquad \lim_{x=0}\frac{x}{e^x-1}=1 \qquad \text{(G-14)}$$

$$\lim_{x=0}\frac{x}{\log_b\left(1+x\right)}=\ln b \qquad \lim_{x=0}\frac{\left(1+x\right)^m-1}{x}=m \qquad \text{(G-15)}$$

$$\lim_{x=0}\frac{\ln\left(1+x\right)}{x}=1 \qquad \lim_{n=\infty}\frac{k^n}{n!}=0 \qquad \text{(G-16)}$$

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Grenzwerte

$$\lim_{x=0}\frac{\sin x}{x}=1 \qquad \lim_{\delta=0}\frac{\left(1+\delta\right)^\mu-1}{\delta}=\mu \qquad \text{(G-17)}$$

$$\lim_{x=0}\frac{\operatorname{tg} x}{x}=1 \qquad \text{(G-18)}$$

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Differentialquotienten

$$y=x^\alpha \qquad \frac{dy}{dx}=\alpha x^{\alpha-1} \qquad y=\sin x \qquad y'=\cos x \qquad \text{(D-1)}$$

$$y=b^x \qquad \frac{dy}{dx}=b^x\ln b \qquad y=\cos x \qquad y'=-\sin x \qquad \text{(D-2)}$$

$$y = e^x \quad y' = e^x \qquad y = \cosh x = \frac{e^x + e^{-x}}{2} \qquad y' = \sinh x \tag{D-3}$$

$$y = \log_b x \qquad y' = \frac{\log_b e}{x} \qquad y = \sinh x \qquad y' = \cosh x \tag{D-4}$$

$$y = \ln x \qquad y' = \frac{1}{x} \tag{D-5}$$

$$\text{Product: } \frac{\mathrm{d}(\mathrm{u} \cdot \mathrm{v})}{\mathrm{dx}} = u \cdot \frac{\mathrm{dv}}{\mathrm{dx}} + v[\cdot]\frac{\mathrm{du}}{\mathrm{dx}} \tag{D-6}$$

$$\text{Quotient: } \frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{u}{v}\right) = \frac{v[\cdot]\mathrm{d}u - u[\cdot]\mathrm{d}v}{v^2} \tag{D-7}$$

$$\frac{d \operatorname{tg} x}{dx} = 1 + \operatorname{tg}^2 x \tag{D-8}$$

$$\frac{d \operatorname{ctg} x}{dx} = -\left(1 + \cot^2 x\right) \tag{D-9}$$

$$\frac{d \sec x}{dx} = \operatorname{tg} x \cdot \sec x \tag{D-10}$$

$$\frac{d \operatorname{cosec} x}{dx} = -\cot x \cdot \operatorname{cosec} \cdot x \tag{D-11}$$

$$\frac{d \arcsin x}{dx} = \frac{1}{\sqrt{1 - x^2}} \tag{D-12}$$

$$\frac{d \arccos x}{dx} = -\frac{1}{\sqrt{1 - x^2}} \tag{D-13}$$

$$\frac{d \operatorname{arctg} x}{dx} = \frac{1}{1 + x^2} \tag{D-14}$$

$$\frac{d \operatorname{arcot} x}{dx} = -\frac{1}{1 + x^2} \tag{D-15}$$

$$\frac{d \operatorname{arcsec} x}{dx} = \frac{1}{x\sqrt{x^2 - 1}} \tag{D-16}$$

$$\frac{d \operatorname{arcosec} x}{dx} = -\frac{1}{x\sqrt{x^2 - 1}} \tag{D-17}$$

Differentialquotienten

Allgemein: Bogenfunctionen

1. $\frac{d \arcsin \varphi(x)}{dx} = \frac{\varphi'(x)}{\sqrt{1-\varphi(x)^2}}$ (D-18)

2. $\frac{d \arccos \varphi(x)}{dx} = \frac{-\varphi'(x)}{\sqrt{1+\varphi(x)^2}}$ (D-19)

3. $\frac{d \operatorname{arctg} \varphi(x)}{dx} = \frac{\varphi'(x)}{1+(\varphi(x))^2}$ (D-20)

4. $\frac{d \operatorname{arccot} \cdot \varphi(x)}{dx} = -\frac{\varphi'(x)}{1+(\varphi(x))^2}$ (D-21)

5. $\frac{d \operatorname{arcsec} \varphi(x)}{dx} = \frac{\varphi'(x)}{\varphi(x)\sqrt{(\varphi(x))^2-1}}$ (D-22)

6. $\frac{d \operatorname{arcosec} \varphi(x)}{dx} = -\frac{\varphi'(x)}{\varphi(x)\sqrt{[\varphi(x)]^2-1}}$ (D-23)

Logarithm. Function: $\frac{d}{dx}\lg \varphi(x) = \frac{\varphi'(x)}{\varphi(x)}$ (D-24)

Funct. von Funct. $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$ z. B.

$$d\left[\frac{1}{b}\ln\left(a+bx\right)\right] = \frac{dx}{a+bx} \qquad d\left[\frac{2\sqrt{a+bx}}{b}\right] = \frac{dx}{\sqrt{a+bx}} \qquad \text{(D-25)}$$

$$d\left[-\frac{1}{b\left(a+bx\right)}\right] = \frac{dx}{\left(a+bx\right)^2} \qquad d\left[\frac{\ln\left(\beta x+\sqrt{\alpha^2+\beta^2x^2}\right)}{\beta}\right] = \frac{dx}{\sqrt{\alpha^2+\beta^2x^2}}$$

(D-26)

$$d\left[\frac{1}{\alpha\cdot\beta}\operatorname{arctg}\frac{\beta x}{\alpha}\right] = \frac{dx}{\alpha^2+\beta^2x^2} \qquad d\left[\frac{1}{\beta}\arcsin\frac{\beta x}{\alpha}\right] = \frac{dx}{\sqrt{\alpha^2-\beta^2x^2}} \qquad \text{(D-27)}$$

$$d\left[\frac{1}{2\alpha\beta}\ln\left(\frac{\alpha+\beta x}{\alpha-\beta x}\right)\right]=\frac{dx}{\alpha^2-\beta^2x^2} \qquad d\left[\frac{\sqrt{a+bx^2}}{b}\right]=\frac{xdx}{\sqrt{a+bx^2}} \tag{D-28}$$

$$d\left[\frac{1}{2b}\ln\left(a+bx^2\right)\right]=\frac{xdx}{a+bx^2} \tag{D-29}$$

$$d\left[\frac{x}{a\sqrt{a+bx^2}}\right]=\frac{dx}{\sqrt{\left(a+bx^2\right)^3}} \qquad d\left[-\frac{1}{b\sqrt{a+bx^2}}\right]=\frac{xdx}{\sqrt{\left(a+bx^2\right)^3}} \tag{D-30}$$

$$d\left[x\ln x-x\right]=\ln xdx \qquad d\left[-\ln\cos u\right]=\operatorname{tg}udu \tag{D-31}$$

$$d\left[\ln\sin u\right]=\cot udu \qquad d\left[\frac{1}{\alpha\beta}\operatorname{arctg}\left(\frac{\beta\operatorname{tg}u}{\alpha}\right)\right]=\frac{du}{\alpha^2\cos^2u+\beta^2\sin^2u} \tag{D-32}$$

$$d\left[\frac{1}{2\alpha\beta}\ln\left(\frac{\alpha+\beta\operatorname{tg}u}{\alpha-\beta\operatorname{tg}u}\right)\right]=\frac{du}{\alpha^2\cos^2u-\beta^2\sin^2u} \tag{D-33}$$

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Differentialquotienten

1.) $z=f(xy);\qquad dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}$ (D-34)

2.) $u=F(xyz)\qquad du=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy+\frac{\partial u}{\partial z}dz$ (D-35)

2a) oder $y=f\left(u,v\right)$ beide abhängig von x (D-36)

so $\frac{dy}{dx}=\frac{\partial f}{\partial u}\cdot u'+\frac{\partial f}{\partial v}\cdot v'$ woraus obiges folgt

3.) $y=f\left[u\left(z\right),v\left(z_1\right)\right]\ldots z\left(x\right),z_1\left(x\right)$

$$\frac{dy}{dx}=\frac{\partial f}{\partial u}\cdot\frac{du}{dx}+\frac{\partial f}{\partial v}\cdot\frac{dv}{dx};\quad \frac{du}{dx}=\frac{du}{dz}\cdot\frac{dz}{dx},\ \frac{dv}{dx}=\frac{dv}{dz_1}\cdot\frac{dz_1}{dx} \tag{D-37}$$

Implicite Functionen

$$F(x,y)=0 \qquad \frac{dy}{dx}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} \tag{D-38}$$

$$\frac{\partial^2 F}{\partial x \partial y}=\frac{\partial^2 F}{\partial y \partial x} \tag{D-39}$$

Höhere Diff. Quot.

Es sei $D(y)=\frac{\mathrm{dy}}{\mathrm{dx}}: D(x^{\mu})=\mu x^{\mu-1}, \quad D^n(x^{\mu})=\mu(\mu-1)(\mu-2)...(\mu-[n-1])x^{\mu-n}$ (D-40)

$$D^n(a+bx)^{\mu}=\mu(\mu-1)...(\mu-[n-1])b^n(a+bx)^{\mu-n} \tag{D-41}$$

$$D^n\frac{1}{a+bx}=\frac{(-1)^n\cdot 1\cdot 2...n\cdot b^n}{(a+bx)^{n+1}} \tag{D-42}$$

$D^n \log x = M\frac{(-1)^{n-1}\cdot 1\cdot 2...(n-1)}{x^n}$ $\quad M$ = Modul des log. Systems (D-43)

$$D^n \log(a+bx)=M\frac{(-1)^{n-1}\cdot 1\cdot 2...(n-1)b^n}{(a+bx)^n} \tag{D-44}$$

$$D^n a^x = a^x(\ln a)^n \tag{D-45}$$

$$D^n e^{\beta x}=\beta^n e^{\beta x} \tag{D-46}$$

$$D^n \sin x=\sin\left(\frac{n}{2}\pi+x\right) \tag{D-47}$$

$$D^n \cos x=\cos\left(\frac{n}{2}\pi+x\right) \tag{D-48}$$

Höhere Diff. Quot. zusammengesetzter Funct.

$$D^n(au+bv)=aD^nu+bD^nv \tag{D-49}$$

$$D^n(u\cdot v)=(n_0)u\cdot D^nv+(n)_1 Du\cdot D^{n-1}v+(n)_2 D^2u\cdot D^{n-2}v+... \tag{D-50}$$

z. B. $D^n \frac{\ln x}{x} = \frac{(-1)^n \cdot 1 \cdot 2 \ldots n}{x^{n+1}} \left[\ln x - \left(\frac{1}{1} + \frac{1}{2} + \ldots + \frac{1}{n} \right) \right]$ (D-51)

$$D^n \sec x = \left[(n)_1 \sec x^{(n-1)} - (n)_3 \sec x^{(n-3)} + (n)_5 \sec x^{(n-5)} - \cdots \right] \operatorname{tg} x + (n)_2 \sec x^{(n-2)} - (n)_4 \sec x^{(n-4)} + (n)_6 \sec x^{(n-6)} - \cdots \quad \text{(D-52)}$$

$$D^n \operatorname{tg} x = \frac{\sin\left(\frac{1}{2} n\pi + x \right)}{\cos x} + \left[(n_1) \operatorname{tg} x^{(n-1)} - (n_3) \operatorname{tg} x^{(n-3)} + \cdots \right] \operatorname{tg} x + (n_2) \operatorname{tg} x^{(n-2)} - (n_4) \operatorname{tg} x^{(n-4)} + \cdots \quad \text{(D-53)}$$

$$D^{n+2} \arcsin x = \frac{(2n+1)\, x \arcsin x^{(n+1)} + n^2 \sin x^{(n)}}{1 - x^2} \quad \text{(D-54)}$$

oder

$$D^{n+1} \arcsin x = \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^n (1-x)^n \sqrt{1-x^2}} \left\{ 1 - \frac{1 \cdot (n)_1 \cdot (1-x)}{2n-1 \cdot (1+x} + \frac{1 \cdot 3 \cdot (n)_2}{(2n-1)(2n-3)} \cdot \left(\frac{1-x}{1+x} \right)^2 - \frac{1 \cdot 3 \cdot 5 (n)_3}{(2n-1)(2n-3)(2n-5)} \left(\frac{1-x}{1+x} \right)^3 + \cdots \right\} \quad \text{(D-55)}$$

$$D^{n+1} \operatorname{arctg} x = \frac{2nx \operatorname{arctg} x^{(n)} + n(n-1) \operatorname{arctg} x^{(n-1)}}{1+x^2} \quad \text{(D-56)}$$

oder

$$D^{n+1} \operatorname{arctg} x = \frac{(-1)^{n-1} \cdot 1 \cdot 2 \cdots (n-1)}{\sqrt{(1+x^2)^n}} \sin\left(n \operatorname{arctg} \frac{1}{x} \right) \quad \text{(D-57)}$$

<u>Höhere partielle Diff. Quot.</u>

$$d^2 z = \frac{\partial^2 z}{\partial^2 x} dx^2 + 2 \frac{\partial^2 z}{\partial x \partial y} dx dy + \frac{\partial^2 z}{\partial^2 y} dy^2 \quad \text{(D-58)}$$

$$d^3 z = \frac{\partial^3 z}{\partial x^3} dx^3 + 3 \frac{\partial^3 z}{\partial^2 x \partial y} dx^2 dy + 3 \frac{\partial^3 z}{\partial x \partial y^2} dx dy^2 + \frac{\partial^3 z}{\partial y^3} dy^3 \quad \text{(D-59)}$$

$$d^n z = (n)_0 \frac{\partial^n z}{\partial x^n} dx^n + (n)_1 \frac{\partial^n z}{\partial x^{n-1} \partial y} dx^{n-1} dy + (n)_2 \frac{\partial^n z}{\partial x^{n-2} \partial y^2} + \cdots$$

$$= \left(\frac{1}{\partial x} dx + \frac{1}{\partial y} dy \right)^n \partial^n z \text{ symbolisch} \qquad \text{(D-60)}$$

Variable:

$$d^n \mu = \left(\frac{1}{\partial x} + \frac{1}{\partial y} + \frac{1}{\partial z} \right)^n \partial^n \mu \qquad n \text{ Variable /. nächste Seite} \qquad \text{(D-61)}$$

<u>Höhere Diff.Quot. impliciter Functionen</u>

$$\frac{\partial^2 f}{\partial x^2} + 2 \frac{\partial^2 f}{\partial x \partial y} \cdot \frac{dy}{dx} + \frac{\partial^2 f}{\partial y^2} \cdot \left(\frac{dy}{dx} \right)^2 + \frac{\partial f}{\partial y} \cdot \frac{d^2 y}{dx^2} = 0 \qquad \text{(D-62)}$$

$$\frac{\partial^3 f}{\partial x^3} + 3 \frac{\partial^3 f}{\partial x^2 \partial y} \cdot \frac{dy}{dx} + \frac{3 \partial^3 f}{\partial x \partial y^2} \left(\frac{dy}{dx} \right)^2 + \frac{\partial^3 f}{\partial y^3} \left(\frac{dy}{dx} \right)^3 + $$
$$+ 3 \frac{\partial^2 f}{\partial x \partial y} \cdot \frac{d^2 y}{dx^2} + 3 \frac{\partial^2 f}{\partial y^2} \cdot \frac{dy}{dx} \cdot \frac{d^2 y}{dx^2} + \frac{\partial f}{\partial y} \frac{d^3 y}{dx^3} = 0 \qquad \text{(D-63)}$$

$\frac{dy}{dx}$ nach früher $\frac{d^2 y}{dx^2}$ unter Einsetzung dieses berechnen u.<nd.> s<iehe> o<ben> schrittweise weiter

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<u>Differential Quotienten.</u>

<u>Differentiale von Reihen</u>: siehe daselbst!

<u>Höhere partielle Differentialquotienten</u>

n unabhängige Veränderliche:

$y = f(x_1 \ldots x_n)$

$$: d^2 y = \left(\frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 + \cdots + \frac{\partial y}{\partial x_n} dx_n \right)^2 \qquad \text{(D64)}$$

$$d^n y = \left(\frac{\partial y}{\partial x_1} dx_1 + \cdots + \frac{\partial y}{\partial x_n} \; dx_n \right)^n \tag{D-65}$$

2 abhängige Variable:

$z = f(xy), \qquad y = \varphi(x) \qquad\qquad \ldots 1.)$

$x = \psi(\lambda), \quad y = \chi(\lambda) \ldots 2.)$

$$: dy = \varphi'(x)\,dx = \chi'(\lambda)\,d\lambda; \quad dx = \psi'(\lambda)\cdot d\lambda, \quad \frac{dy}{dx} = \frac{\chi'(\lambda)}{\psi'(\lambda)} \tag{D-66}$$

$$d^2 z = \left(\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \right)^2 + \frac{\partial z}{\partial x} d(dx) + \frac{\partial z}{\partial y} d(dy) \tag{D-67}$$

1.) $$d(dx) = 0 \tag{D-68}$$

$$d(dy) = \varphi''(x)\,dx^2 \tag{D-69}$$

$$d^2 z = \left(\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \right)^2 + \varphi''(x)\,dx^2 \cdot \frac{\partial z}{\partial y} \tag{D-70}$$

2.) $$d(dx) = d^2 x = \varphi''(\lambda)\,d\lambda^2 \tag{D-71}$$

$$d(dy) = d^2 y = \chi''(\lambda)\,d\lambda^2 \tag{D-72}$$

$$d^2 z = \left(\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \right)^2 + \frac{\partial z}{\partial x} d^2 x + \frac{\partial z}{\partial y} d^2 y \tag{D-73}$$

$$d^3 z = (dz)^3 + \frac{\partial^2 z}{\partial x^2} d\left(dx^2\right) + 2\frac{\partial^2 z}{\partial x \partial y} d(dxdy) + \frac{\partial^2 z}{\partial y^2} d(dy) \tag{D-74}$$

wobei: $$d\left(dx^2\right) = 2dx \cdot d^2 x = 2\psi'(\lambda)\psi''(\lambda)\,d\lambda^3 \tag{D-75}$$

<u>Ausnahme:</u> Ist $$x = \varphi(\lambda) = a\lambda + b \tag{D-76}$$

$$y = \chi(\lambda) = \alpha\lambda + \beta \text{ so ist} \tag{D-77}$$

$$d^3 z = (dz)^3 \text{ da } \psi' = a, \;\; \psi'' \;\; \text{u.<und>f.<folgende>} = 0 \tag{D-78}$$

$$\chi' = \alpha \quad \chi'' \quad \text{“ “} = 0 \tag{D-79}$$

Diff. mehrerer Functionen einer Veränderlichen.

$$y=f\left(x_1 \ldots\ldots x_n\right), \qquad x_1=f_1\left(x\right)\ldots\ldots x_n=f_n\left(x\right)$$

$$\frac{dy}{dx}=\frac{\partial f_1}{\partial x_1}x_1'+\frac{\partial f_2}{\partial x_2}x_2'+\cdots+\frac{\partial f_n}{\partial x_n}x_n' \tag{D-80}$$

$$\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2+\frac{\partial f}{\partial x_1}x_1''+\ldots\frac{\partial f}{\partial x_n}x_n'' \tag{D-81}$$

$+F\left(xy\right)=0$

$$\frac{d^2y}{dx^2}=-\frac{\frac{\partial^2F}{\partial x^2}\left(\frac{\partial F}{\partial y}\right)^2-2\frac{\partial^2F}{\partial x\partial y}\frac{\partial F}{\partial x}\frac{\partial F}{\partial y}+\frac{\partial^2F}{\partial y^2}\left(\frac{\partial F}{\partial x}\right)^2}{\left(\frac{\partial F}{\partial y}\right)^3} \tag{D-82}$$

$$F\left(xyz\right)=0 \tag{D-83}$$

$$p=\frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}, \qquad \frac{\partial z}{\partial y}=-\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}=q \tag{D-84}$$

$$\frac{\partial^2z}{\partial x^2}=r=-\frac{\frac{\partial^2F}{\partial x^2}+2\frac{\partial^2F}{\partial x\partial z}p+\frac{\partial^2F}{\partial z^2}p^2}{\frac{\partial F}{\partial z}} \tag{D-85}$$

$$\frac{\partial p}{\partial y}=\frac{\partial q}{\partial x}=\frac{\partial^2z}{\partial x\partial y}=s=-\frac{\frac{\partial^2F}{\partial x\partial y}+\frac{\partial^2F}{\partial x\partial z}q+\frac{\partial^2p}{\partial y\partial z}p+\frac{\partial^2F}{\partial z^2}p\cdot q}{\frac{\partial F}{\partial z}} \tag{D-86}$$

$$\frac{\partial^2z}{\partial y^2}=t=-\frac{\frac{\partial^2F}{\partial y^2}+2\frac{\partial^2F}{\partial y\partial z}q+\frac{\partial^2F}{\partial z^2}q^2}{\frac{\partial F}{\partial z}} \tag{D-87}$$

Einführung neuer Veränderlicher.

$$\left.\begin{array}{r} y=f(x) \\ F(xy)=0 \end{array}\right\} x=\psi(t) \text{ gesetzt. Sodann: } y=f\left(\psi\left(t\right)\right)=\chi\left(t\right)$$

$$\frac{dy}{dx}=\frac{\chi'}{\psi'} \tag{D-88}$$

$$\frac{d^2y}{dx^2}=\frac{\psi'\chi''-\chi'\psi''}{\psi'^3} \tag{D-89}$$

$$\frac{d^3y}{dx^3}=\frac{1}{\psi'}\frac{d\left(\frac{\psi'\chi''-\chi'\psi''}{\psi'^3}\right)}{dt} \tag{D-90}$$

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Tangenten u. Normalen

$y=f\left(x\right)$ geg. Curve

$$\eta-y=\frac{dy}{dx}\left(\xi-x\right) \quad \ldots\text{Tg.} \tag{TN-1}$$

$$\eta-y=-\frac{1}{\frac{dy}{dx}}\left(\xi-x\right) \quad \text{Norm.} \tag{TN-2}$$

$F\left(xy\right)=0$ geg. Curve

$$\frac{\partial F}{\partial x}\left(\xi-x\right)+\frac{\partial F}{\partial y}\left(\eta-y\right)=0 \quad \text{Tang.} \tag{TN-3}$$

$$\frac{\partial F}{\partial y}\left(\xi-x\right)-\frac{\partial F}{\partial x}\left(\eta-y\right)=0 \quad \text{Norm.} \tag{TN-4}$$

$\left.\begin{array}{l} x=\varphi(t) \\ y=\psi(t) \end{array}\right\}$ geg. Curve

$$\varphi'\left(t\right)\left(\eta-y\right)-\psi'\left(t\right)\left(\xi-x\right)=0 \quad \text{Tang.} \tag{TN-5}$$

$$\psi'\left(t\right)\left(\eta-y\right)+\varphi'\left(t\right)\left(\xi-x\right)=0 \quad \text{Norm.} \tag{TN-6}$$