Differential Quotienten.

Differentiale von Reihen: siehe daselbst!

Höhere partielle Differentialquotienten

n unabhängige Veränderliche:

$y = f(x_1 \ldots x_n)$

$$: d^2y = \left(\frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 + \cdots + \frac{\partial y}{\partial x_n} dx_n \right)^2 \tag{D-64}$$

$$d^n y = \left(\frac{\partial y}{\partial x_1} dx_1 + \cdots + \frac{\partial y}{\partial x_n} dx_n \right)^n \tag{D-65}$$

2 abhängige Variable:

$z = f(xy), \quad y = \varphi(x) \quad \ldots 1.)$

$x = \psi(\lambda), \quad y = \chi(\lambda) \ldots 2.)$

$$: dy = \varphi'(x)dx = \chi'(\lambda)d\lambda; \quad dx = \psi'(\lambda) \cdot d\lambda, \quad \frac{dy}{dx} = \frac{\chi'(\lambda)}{\psi'(\lambda)} \tag{D-66}$$

$$d^2z = \left(\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \right)^2 + \frac{\partial z}{\partial x} d(dx) + \frac{\partial z}{\partial y} d(dy) \tag{D-67}$$

1.) $$d(dx) = 0 \tag{D-68}$$

$$d(dy) = \varphi''(x) dx^2 \tag{D-69}$$

$$d^2z = \left(\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \right)^2 + \varphi''(x) dx^2 \cdot \frac{\partial z}{\partial y} \tag{D-70}$$

2.) $$d(dx) = d^2x = \varphi''(\lambda) d\lambda^2 \tag{D-71}$$

$$d(dy) = d^2y = \chi''(\lambda) d\lambda^2 \tag{D-72}$$

$$d^2z = \left(\frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \right)^2 + \frac{\partial z}{\partial x} d^2x + \frac{\partial z}{\partial y} d^2y \tag{D-73}$$

$$d^3z = (dz)^3 + \frac{\partial^2 z}{\partial x^2} d\left(dx^2\right) + 2\frac{\partial^2 z}{\partial x \partial y} d(dxdy) + \frac{\partial^2 z}{\partial y^2} d(dy) \tag{D-74}$$

wobei: $$d\left(dx^2\right) = 2dx \cdot d^2x = 2\psi'(\lambda)\psi''(\lambda)d\lambda^3 \tag{D-75}$$

<u>Ausnahme:</u> Ist $$x = \varphi(\lambda) = a\lambda + b \tag{D-76}$$

$$y = \chi(\lambda) = \alpha\lambda + \beta \text{ so ist} \tag{D-77}$$

$$d^3z = (dz)^3 \text{ da} \qquad \psi' = a, \ \psi'' \text{ u.<und>f.<folgende>} = 0 \tag{D-78}$$

$$\chi' = \alpha \quad \chi'' \quad \text{“ “} = 0 \tag{D-79}$$

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<u>Diff. mehrerer Functionen einer Veränderlichen.</u>

$$y = f\left(x_1 \ldots \ldots x_n\right), \qquad x_1 = f_1(x) \ldots \ldots x_n = f_n(x)$$

$$\frac{dy}{dx} = \frac{\partial f_1}{\partial x_1} x_1' + \frac{\partial f_2}{\partial x_2} x_2' + \ldots + \frac{\partial f_n}{\partial x_n} x_n' \tag{D-80}$$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 + \frac{\partial f}{\partial x_1} x_1'' + \ldots \frac{\partial f}{\partial x_n} x_n'' \tag{D-81}$$

$$\underline{\underline{+F(xy) = 0}}$$

$$\frac{d^2y}{dx^2} = -\frac{\dfrac{\partial^2 F}{\partial x^2}\left(\dfrac{\partial F}{\partial y}\right)^2 - 2\dfrac{\partial^2 F}{\partial x \partial y}\dfrac{\partial F}{\partial x}\dfrac{\partial F}{\partial y} + \dfrac{\partial^2 F}{\partial y^2}\left(\dfrac{\partial F}{\partial x}\right)^2}{\left(\dfrac{\partial F}{\partial y}\right)^3} \tag{D-82}$$

$$F(xyz) = 0 \tag{D-83}$$

$$p = \frac{\partial z}{\partial x} = -\frac{\dfrac{\partial F}{\partial x}}{\dfrac{\partial F}{\partial z}}, \qquad \frac{\partial z}{\partial y} = -\frac{\dfrac{\partial F}{\partial y}}{\dfrac{\partial F}{\partial z}} = q \tag{D-84}$$

$$\frac{\partial^2 z}{\partial x^2} = r = -\frac{\frac{\partial^2 F}{\partial x^2} + 2\frac{\partial^2 F}{\partial x \partial z}p + \frac{\partial^2 F}{\partial z^2}p^2}{\frac{\partial F}{\partial z}} \tag{D-85}$$

$$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x} = \frac{\partial^2 z}{\partial x \partial y} = s = -\frac{\frac{\partial^2 F}{\partial x \partial y} + \frac{\partial^2 F}{\partial x \partial z}q + \frac{\partial^2 p}{\partial y \partial z}p + \frac{\partial^2 F}{\partial z^2}p \cdot q}{\frac{\partial F}{\partial z}} \tag{D-86}$$

$$\frac{\partial^2 z}{\partial y^2} = t = -\frac{\frac{\partial^2 F}{\partial y^2} + 2\frac{\partial^2 F}{\partial y \partial z}q + \frac{\partial^2 F}{\partial z^2}q^2}{\frac{\partial F}{\partial z}} \tag{D-87}$$

<u>Einführung neuer Veränderlicher.</u>

$\left.\begin{array}{r} y = f(x) \\ F(xy) = 0 \end{array}\right\} x = \psi(t)$ gesetzt. Sodann: $y = f\left(\psi\left(t\right)\right) = \chi\left(t\right)$

$$\frac{dy}{dx} = \frac{\chi'}{\psi'} \tag{D-88}$$

$$\frac{d^2 y}{dx^2} = \frac{\psi'\chi'' - \chi'\psi''}{\psi'^3} \tag{D-89}$$

$$\frac{d^3 y}{dx^3} = \frac{1}{\psi'}\frac{d\left(\frac{\psi'\chi'' - \chi'\psi''}{\psi'^3}\right)}{dt} \tag{D-90}$$