Differentialquotienten

1.) $z = f(xy);\quad dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}$ (D-34)

2.) $u = F(xyz)\quad du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$ (D-35)

2a) oder $y = f(u,v)$ beide abhängig von x (D-36)

so $\frac{dy}{dx} = \frac{\partial f}{\partial u}\cdot u' + \frac{\partial f}{\partial v}\cdot v'$ woraus obiges folgt

3.) $y = f\left[u(z),v(z_1)\right]\mathrm{K}\ z(x),z_1(x)$

$$\frac{dy}{dx} = \frac{\partial f}{\partial u}\cdot\frac{du}{dx} + \frac{\partial f}{\partial v}\cdot\frac{dv}{dx};\quad \frac{du}{dx} = \frac{du}{dz}\cdot\frac{dz}{dx},\ \frac{dv}{dx} = \frac{dv}{dz_1}\cdot\frac{dz_1}{dx} \qquad \text{(D-37)}$$

Implicite Functionen

$$F(x,y) = 0\quad \frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} \qquad \text{(D-38)}$$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x} \qquad \text{(D-39)}$$

Höhere Diff. Quot.

Es sei $D(y) = \frac{dy}{dx} : D(x^\mu) = \mu\, x^{\mu-1},\quad D^n(x^\mu) = \mu(\mu-1)(\mu-2)\ldots(\mu-[n-1])x^{\mu-n}$ (D-40)

$$D^n(a+b\,x)^\mu = \mu(\mu-1)\ldots(\mu-[n-1])b^n\,(a+b\,x)^{\mu-n} \qquad \text{(D-41)}$$

$$D^n\,\frac{1}{a+b\,x} = \frac{(-1)^n\cdot 1\cdot 2\ldots n\cdot b^n}{(a+b\,x)^{n+1}} \qquad \text{(D-42)}$$

$$D^n \log x = M\,\frac{(-1)^{n-1}\cdot 1\cdot 2\ldots(n-1)}{x^n} \qquad M = \text{Modul des log. Systems} \qquad \text{(D-43)}$$

$$D^n \log(a+b\,x) = M\,\frac{(-1)^{n-1}\cdot 1\cdot 2\ldots(n-1)\,b^n}{(a+b\,x)^n} \qquad \text{(D-44)}$$

$$D^n a^x = a^x\,(\ln a)^n \qquad \text{(D-45)}$$

$$D^n e^{\beta x} = \beta^n e^{\beta x} \qquad \text{(D-46)}$$

$$D^n \sin x = \sin\left(\frac{n}{2}\pi + x\right) \qquad \text{(D-47)}$$

$$D^n \cos x = \cos\left(\frac{n}{2}\pi + x\right) \qquad \text{(D-48)}$$

<u>Höhere Diff. Quot. zusammengesetzter Funct.</u>

$$D^n(a\,u + b\,v) = a\,D^n u + b\,D^n v \qquad \text{(D-49)}$$

$$D^n(u\cdot v) = (n_0)u\cdot D^n v + (n)_1\,D\,u\cdot D^{n-1}v + (n)_2\,D^2 u\cdot D^{n-2}v + \ldots \qquad \text{(D-50)}$$

z. B. $D^n\,\frac{\ln x}{x} = \frac{(-1)^n\cdot 1\cdot 2\ldots n}{x^{n+1}}\left[\ln x - \left(\frac{1}{1}+\frac{1}{2}+\ldots+\frac{1}{n}\right)\right]$ (D-51)

$$\begin{aligned} D^n \sec x = {} & \left[(n)_1 \sec x^{(n-1)} - (n)_3 \sec x^{(n-3)} + (n)_5 \sec x^{(n-5)} - \ldots\ \right]\operatorname{tg} x \\ & + (n)_2 \sec x^{(n-2)} - (n)_4 \sec x^{(n-4)} + (n)_6 \sec x^{(n-6)} - \ldots \end{aligned} \qquad \text{(D-52)}$$

$$\begin{aligned} D^n \operatorname{tg} x = {} & \frac{\sin\left(\frac{1}{2}n\pi + x\right)}{\cos x} + \left[(n_1)\operatorname{tg} x^{(n-1)} - (n_3)\operatorname{tg} x^{(n-3)} + \ldots\ \right]\operatorname{tg} x \\ & + (n_2)\operatorname{tg} x^{(n-2)} - (n_4)\operatorname{tg} x^{(n-4)} + \ldots \end{aligned} \qquad \text{(D-53)}$$

$$D^{n+2} \arcsin x = \frac{(2n+1)\,x \arcsin x^{(n+1)} + n^2 \arcsin x^{(n)}}{1-x^2} \qquad \text{(D-54)}$$

oder

$$D^{n+1}\arcsin x = \frac{1\cdot 3\cdot 5\ldots(2n-1)}{2^n\,(1-x)^n\,\sqrt{1-x^2}}\left\{1-\frac{1\cdot(n)_1\cdot(1-x)}{2n-1\cdot(1+x}+\frac{1\cdot 3\cdot(n)_2}{(2n-1)(2n-3)}\cdot\left(\frac{1-x}{1+x}\right)^2\right. \qquad \text{(D-55)}$$

$$\left.-\frac{1\cdot 3\cdot 5\,(n)_3}{(2n-1)(2n-3)(2n-5)}\left(\frac{1-x}{1+x}\right)^3+\ldots\right\}$$

$$D^{n+1}\operatorname{arctg} x = -\frac{2nx\operatorname{arctg} x^{(n)}+n(n-1)\operatorname{arctg} x^{(n-1)}}{1+x^2} \qquad \text{(D-56)}$$

oder

$$D^{n+1}\operatorname{arctg} x = \frac{(-1)^{n-1}\cdot 1\cdot 2\cdot\ldots\,(n-1)}{\sqrt{(1+x^2)^n}}\sin\left(n\operatorname{arctg}\frac{1}{x}\right) \qquad \text{(D-57)}$$

<u>Höhere partielle Diff. Quot.</u>

$$d^2z = \frac{\partial^2 z}{\partial^2 x}dx^2 + 2\frac{\partial^2 z}{\partial x\partial y}dxdy + \frac{\partial^2 z}{\partial^2 y}dy^2 \qquad \text{(D-58)}$$

$$d^3z = \frac{\partial^3 z}{\partial x^3}dx^3 + 3\frac{\partial^3 z}{\partial^2 x\partial y}dx^2dy + 3\frac{\partial^3 z}{\partial x\partial y^2}dxdy^2 + \frac{\partial^3 z}{\partial y^3}dy^3 \qquad \text{(D-59)}$$

$$d^nz = (n)_0\frac{\partial^n z}{\partial x^n}dx^n + (n)_1\frac{\partial^n z}{\partial x^{n-1}\partial y}dx^{n-1}dy + (n)_2\frac{\partial^n z}{\partial x^{n-2}\partial y^2} + \ldots$$

$$= \left(\frac{1}{\partial x}dx + \frac{1}{\partial y}dy\right)^n \partial^n z \ \text{ symbolisch} \qquad \text{(D-60)}$$

Variable:

$$d^n\mu = \left(\frac{1}{\partial x}+\frac{1}{\partial y}+\frac{1}{\partial z}\right)^n \partial^n\mu \ \ n \ \text{ Variable /. nächste Seite} \qquad \text{(D-61)}$$

<u>Höhere Diff.Quot. impliciter Functionen</u>

$$\frac{\partial^2 f}{\partial x^2} + 2\frac{\partial^2 f}{\partial x\partial y}\cdot\frac{dy}{dx} + \frac{\partial^2 f}{\partial y^2}\cdot\left(\frac{dy}{dx}\right)^2 + \frac{\partial f}{\partial y}\cdot\frac{d^2 y}{dx^2} = 0 \qquad \text{(D-62)}$$

$$\frac{\partial^3 f}{\partial x^3} + 3\frac{\partial^3 f}{\partial x^2 \partial y} \cdot \frac{dy}{dx} + \frac{3\partial^3 f}{\partial x \partial y^2}\left(\frac{dy}{dx}\right)^2 + \frac{\partial^3 f}{\partial y^3}\left(\frac{dy}{dx}\right)^3 + \\ + 3\frac{\partial^2 f}{\partial x \partial y} \cdot \frac{d^2 y}{dx^2} + 3\frac{\partial^2 f}{\partial y^2} \cdot \frac{dy}{dx} \cdot \frac{d^2 y}{dx^2} + \frac{\partial f}{\partial y}\frac{d^3 y}{dx^3} = 0 \tag{D-63}$$

$\frac{dy}{dx}$ nach früher $\frac{d^2 y}{dx^2}$ unter Einsetzung dieses berechnen u.<nd.> s<iehe> o<ben> schrittweise weiter